## Triangle Congruences

Congruent triangles are triangles that have the same size and shape. This means that the corresponding sides are equal and the corresponding angles are equal.
**Note: not all the six corresponding elements of both triangles must be found to determine that the two triangles are congruent. There are 5 conditions for determining that two triangles are congruent that we could use. They are the SSS, SAS, ASA, AAS, and HL congruence properties.

## Rules for Triangle Congruency



Our Task:We are going to explore why SSA is not a condition that we could use to explain that two triangles are congruent.

## Step 1: Open Geogebra Classic - WWW.geogebra.org/classic

Step 2: Using the segment tools create a Ray- Go to ${ }^{\circ}$ click on it then click on the option that says Ray. Click in your work area and you should see a line appear, then go further down the line and click again.

Step 3:Create an angle using the segment tools- Go to $\left\lfloor\mathscr{L}^{\circ}\right.$. the angle with the given measure. Going in Counter Clockwise click B then A. Type in 40 degrees and hit okay.


Step 4: Make a line along the 40 degrees- Go to $\left[\square^{\circ}\right.$ click on the ray button and make a ray connected to the 40 degree angle.

Step 5: Create a line segment connected to the 40 degree angle that is 5 units in length. Go to $\lfloor\cdot$ click on segment with given length, go to point A click it and type 5 units then hit okay. You can go to this tool to be able to move the segment around.


Step 6: Make a Circle with the center being A- Go to $\odot^{\circ}$ click on the circle with Center through Point. Create a circle from point A to point D. Where the circle intersects with the ray $A C$ put a point of intersection. Go to hit intersect, then click on the intersection that we created. You can hide the circle and the line segment if you want to make the diagram look less cluttered.

Step 7: Create the last Side of the triangle- From point E create a line segment with a given length of 4. Go to $\left[{ }^{\circ}\right.$ click on segment with given length, go to point $E$ and enter the length 4 units.


Step 8: Move the line segment EF around to try and close the triangle. What do you notice?

Step 9: Create a circle with the center being E-Go to ${ }^{\circ}$ click on the circle with Center through Point. Create a circle from point E to F. Notice that there are two intersections formed. Create the points of the intersection. Then draw line segments from $E$ to $G$ and then E to H .


What are our conclusions about why SSA cannot be used to say two triangles are congruent to each other?

Let's make a new triangle using a different angle measurement You don't have to clear your screen, just hit $\stackrel{\leftrightarrow}{ }$ and you can move the area around on your screen to give you more blank space to work with.

We are repeating most steps from before but changing the angle measures and side lengths

Step 1: Create a Ray- Go to $\left[\sigma^{0}\right.$ click on it then click on the option that says Ray. Step 2: Go to $\lfloor<$. in Counter Clockwise click J then I. Type in 30 degrees
Step 3: Make a line along the 30 degrees- Go to $[\checkmark$ click on the ray button and make a ray connected to the 30 degree angle.


Step 4: Create a line segment 6 units in length. From point I.
Step 5: Make a Circle with the center being I-Go to $\odot$ click on the circle with Center through Point. Create a circle from point I to point K. Where the circle intersects with the ray put a point of intersection. Go to hit intersect, then click on the intersection that we created. Now we have point $L$.
Step 6: Create the last Side of the triangle- From point L create a line segment with a given length of 3 . Go to $[\circ$ click on segment with given length, go to point $L$ and enter the length 3 units.


Step 7: Move the line segment CF around to try and close the triangle. What do you notice?

Step 8: Create a circle with the center being L- Go to click on the circle with Center through Point. Create a circle from point $L$ to $M$. Notice where an intersection is made and mark that point. Now we have Point N.


If we said that SSA isn't a theorem used to prove two triangles congruent then why does this work?

Let's explore further.
First connect the Triangle using segments
Then find the missing angles.
What do you notice about the other angles? What theorem is this triangle a demonstration of?


Challenge question


Each of these five triangles has two sides and one angle that corresponds to Triangle ABC.

1. Identify which triangles are congruent to triangle ABC
2. Consider the triangles that you said were congruent to $\triangle A B C$. What kinds of things were similar about the way those angles and side lengths were arranged compared to $\triangle \mathrm{ABC}$ ?
3. Consider the triangles that you said were NOT congruent to $\triangle A B C$. What kinds of things were similar about the way those angles and side lengths were arranged compared to $\triangle A B C$ ?
